Kuwait University   Math.101   November 2, 1995.
Mathematics Department   First Exam.   Duration: 75 minutes

Answer the following questions: (10 points for each question)

Calculators are not allowed

1. (a) Evaluate each of the following limits, if it exists

   (i) $\displaystyle \lim_{x \to 1} \frac{x^2 + 2x - 3}{x\,|x - 1|}$ (ii) $\displaystyle \lim_{x \to 1} \frac{\sin(x - 1)}{x^2 - 1}$

   (b) Find the following limit, if it exists .

$$\lim_{x \to 0} x^{\frac{2}{3}} \sin \frac{1}{x}$$

2. (a) Find the vertical and horizontal asymptotes if any, of the function

$$f(x) = \frac{4x^2 - 1}{(2x + 1)\sqrt{4x^2 + 3}}$$

   (b) Use the Intermediate value Theorem to show that the function

$$f(x) = x^3 + 2x + 1$$

has a real root.

3. (a) Let

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{\sqrt{x} - 1} & \text{if } x \neq 1 \\[2mm] K & \text{if } x = 1 \end{cases}$$

Find the value of K for which $f(x)$ is continuous at $x = 1$.

   (b) Use the definition of the derivative to find $f'(x)$ where $x > -1$, for the function

$$f(x) = \sqrt{x + 1}$$

   (a) Let

$$f(x) = x^{\frac{2}{3}} + a.$$

Show that $f(x)$ has a cusp at the point $(0, a)$.

   (b) Find an equation of the normal line to the graph of the function

$$f(x) = \frac{x \sec x}{x + 1} + 2$$

at $x = 0$

(*Good Luck*)